

Modular Arithmetic: Multiplication, Division & Binary Operations ($\oplus$, $\otimes$)

This exercise set covers solving equations that involve multiplication and division within a given modulus, and evaluating binary operations defined by symbols such as $\oplus$ and $\otimes$ using Cayley (operation) tables. Work through Section A first, then complete the tables in Section B before attempting the questions that follow each one. **Perfect for students preparing for WAEC, NECO, and JAMB Mathematics.**

Section A — Solving Multiplication & Division Equations in Modular Arithmetic

1. $4x \equiv 2 \pmod{7}$
2. $6x \equiv 3 \pmod{9}$
3. $11 \div 3 \pmod{7}$
4. $9 \div 2 \pmod{7}$
5. $5x \equiv 4 \pmod{6}$
6. $13 \div 4 \pmod{7}$
7. $7x \equiv 3 \pmod{10}$

Section B — Binary Operations & Cayley Tables (mod 5)

1. Copy and complete the tables for addition ($\oplus$) and multiplication ($\otimes$) in mod 5, on the set $\{1, 2, 3, 4\}$.

Addition table ($\oplus$)

$\oplus$	1	2	3	4
1	2		4	
2	3	4		1
3	4			2
4	0	1	2	

Multiplication table ($\otimes$)

$\otimes$	1	2	3	4
1	1			4
2		4		3

3	3		4	
4	4		2	1

(a) Using the tables, evaluate $(3 \otimes 3) \oplus (2 \otimes 2)$.

(b) Find m , such that $m \otimes m = m \oplus m$.

2. Copy and complete the addition ($\oplus$) and multiplication ($\otimes$) tables mod 5, on the set $\{2, 3, 4\}$.

Addition table ($\oplus$)

$\oplus$;	2	3	4
2			
3			
4			

Multiplication table ($\otimes$)

$\otimes$;	2	3	4
2			
3			
4			

(b) Use the tables to solve the equation $4 \otimes e \oplus 2 \equiv 1 \pmod{5}$.

(c) Find n if $4 \oplus (n \otimes 2) \equiv 3 \pmod{5}$.

3. Copy and complete the multiplication table for modulo 5 on the set $\{1, 2, 3, 4\}$ below.

$\otimes$;	1	2	3	4
1	1			
2		4	1	
3	3			2
4		3		1

From the table:

(a) Solve the expression $2n \otimes 4 = 3$.

(b) Find the value of n for which $2 \otimes (3 \otimes n) = 2$.

4. Copy and complete the subtraction table ($\ominus$) for modulo 5 on the set $\{1, 2, 3, 4\}$, where $a \ominus b$ means $(a - b) \pmod{5}$.

$\ominus$;	1	2	3	4
1	0		3	2

2	1	0		3
3		1	0	
4	3		1	0

(a) Using the table, evaluate $(3 \blacksquare 1) \oplus (2 \blacksquare 4)$.

(b) Solve for x : $x \blacksquare 3 \equiv 1 \pmod{5}$.

Note: results of 0 correspond to the modulus itself (5) where relevant.

ANSWERS

Section A

1. $x = 4$

2. $x = 2, 5, 8$

3. $x = 6$

4. $x = 1$

5. $x = 2$

6. $x = 5$

7. $x = 9$

Section B

1. Completed tables:

Addition ($\oplus$)

$\oplus$	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

Multiplication ($\otimes$)

$\otimes$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2

4	4	3	2	1
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(a) $(3 \otimes 3) \oplus (2 \otimes 2) = 4 \oplus 4 = 3$

(b) $m = 2$

2. Tables mirror the mod 5 operations restricted to $\{2, 3, 4\}$ (values may fall outside this set, e.g. 0 or 1, since the table is still worked mod 5).

(b) $4e + 2 \equiv 1 \pmod{5} \Rightarrow 4e \equiv 4 \pmod{5} \Rightarrow e = 1$

(c) $4 + 2n \equiv 3 \pmod{5} \Rightarrow 2n \equiv 4 \pmod{5} \Rightarrow n = 2$

3. Completed table:

$\otimes$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

(a) $8n \equiv 3 \pmod{5} \Rightarrow 3n \equiv 3 \pmod{5} \Rightarrow n = 1$

(b) $3 \otimes n = 1 \Rightarrow n = 2$

4. Completed table:

$\ominus$	1	2	3	4
1	0	4	3	2
2	1	0	4	3
3	2	1	0	4
4	3	2	1	0

(a) $(3 \ominus 1) \oplus (2 \ominus 4) = 2 \oplus 3 = 0$

(b) $x \equiv 4 \pmod{5} \Rightarrow x = 4$