Topic: Set Operations — Union, Intersection, Complement and Cardinality of a Set

Worked Examples

Example A

If ε = {positive integers less than 20}, list the elements of the following sets:

(a) A = {x : x > 5}

Solution: A = {6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19}

(b) B = {x : x is even}

Solution: B = {2, 4, 6, 8, 10, 12, 14, 16, 18}

(c) B = {x : 10 < x ≤ 14}

Solution: B = {11, 12, 13, 14}

(d) A = {x : 10 < x < 18 and x is odd}

Solution: A = {11, 13, 15, 17}

(e) C = {x : x is a prime factor of 21}

Solution: C = {3, 7}

Example B

Given the subsets of a universal set: X = {a, e, m, p}; Y = {m, p, o, u}; Z = {l, m, n, o, p, q, r, s, t, u}. Find:

(a) X ∪ Y

Solution: X ∪ Y = {a, e, m, o, p, u}

(b) X ∩ Y

Solution: X ∩ Y = {m, p}

(c) (X ∪ Y) ∩ (X ∩ Y)

Solution: Since X ∩ Y ⊆ X ∪ Y, the result is X ∩ Y = {m, p}

(d) Y ∩ Z

Solution: Y ∩ Z = {m, o, p, u}

(e) X ∪ (Y ∩ Z)

Solution: X ∪ (Y ∩ Z) = {a, e, m, p} ∪ {m, o, p, u} = {a, e, m, o, p, u}

Example C

Given that U = {1,2,3,4,5,6,7,8,9,10}, P = {1,3,4,6,8,9,10}, Q = {1,3,4,5,6,9}, R = {3,4,5,6,10}. Find the elements of P ∩ Q ∩ R′, where R′ is the complement of R.

Solution: R′ = U − R = {1, 2, 7, 8, 9}. P ∩ Q = {1, 3, 4, 6, 9}. P ∩ Q ∩ R′ = {1, 3, 4, 6, 9} ∩ {1, 2, 7, 8, 9} = {1, 9}

Example D

The universal set ε is the set of all integers. P, Q and R are subsets of ε defined by: P = {x : x ≤ 2}, Q = {x : −7 < x < 15}, R = {x : −2 ≤ x < 19}. Find:

(i) P ∩ Q

Solution: $P \cap Q = \{x : -7 < x \leq 2\}$

(ii) $P \cap (Q \cup R')$

Solution: $R' = \{x : x < -2 \text{ or } x \geq 19\}$. $Q \cup R' = \{x : x < 15 \text{ or } x \geq 19\}$. *Since every value in* P $(x \leq 2)$ *already satisfies* $x < 15$, $P \cap (Q \cup R') = P = \{x : x \leq 2\}$

Section A: General Set Operations

1. Given that ε = {whole numbers from 1 to 25}, A = {x : x is a multiple of 3} and B = {x : x is a multiple of 5}, list the elements of (i) A (ii) B (iii) $A \cap B$ (iv) $A \cup B$ (v) A'.

2. The sets P, Q and R are subsets of ε = {1,2,3,...,15} defined as P = {2,3,5,7,11,13}, Q = {1,3,5,7,9,11,13,15}, R = {2,4,6,8,10,12,14}. Find (i) $P \cap Q$ (ii) $Q \cap R$ (iii) $P \cup R$ (iv) $(P \cap Q)'$ (v) $n(P \cup Q \cup R)$.

3. In a class of 40 students, 25 offer Mathematics, 20 offer Physics, and 12 offer both subjects. Using set notation, find how many students offer (i) Mathematics only (ii) Physics only (iii) neither subject.

4. If ε is the set of all integers, X = {x : $-5 \leq x < 10$} and Y = {x : $0 < x \leq 12$}, find (i) $X \cap Y$ (ii) $X \cup Y$ (iii) X'.

5. Given that A = {x : x is a factor of 24} and B = {x : x is a factor of 36}, find (i) $A \cap B$ (ii) $A \cup B$ (iii) $n(A \cap B)$.

6. In a survey of 50 students, n(ε) = 50. If n(A) = 28, n(B) = 25 and $n(A \cap B) = 15$, find (i) $n(A \cup B)$ (ii) the number of students in neither A nor B.

7. Two sets P and Q are such that n(P) = 20, n(Q) = 15 and $n(P \cup Q) = 28$. Find $n(P \cap Q)$, and state whether P and Q are disjoint, giving a reason.

Section B: Universal Set of Integers / Positive Integers (WAEC-style)

These questions follow the WAEC pattern where the universal set μ (or ε) is defined as the set of all integers, and a subset is restricted to positive integers only.

1. The universal set μ is the set of all integers. X = {x : x is a positive integer, x < 12} and Y = {x : x is a positive even integer, x ≤ 20}. Find (i) X ∩ Y (ii) X ∪ Y (iii) X′.

2. μ = {set of all integers}. A = {x : −3 ≤ x < 8} and B = {x : x is a positive integer}. Find (i) A ∩ B (ii) n(A ∩ B).

3. μ = {set of all integers}. P = {x : x is a positive integer, x ≤ 9} and Q = {x : x is a positive integer, multiple of 3, x ≤ 15}. Find (i) P ∩ Q (ii) P ∩ Q′ (iii) n(P ∩ Q′).

4. The universal set μ = {x : x is an integer, −10 ≤ x ≤ 10}. R = {x : x is a positive integer}. Find (i) R′ (ii) n(R′).

5. μ = {set of all integers}. A = {x : x is a positive integer, multiple of 4, x ≤ 40} and B = {x : x is a positive integer, multiple of 6, x ≤ 40}. Find (i) A ∩ B (ii) n(A ∩ B).

6. μ = {set of all integers}. P = {x : x is a positive integer and x^2 ≤ 50}. List the elements of P and find n(P).

7. μ = {set of all integers}. A = {x : x is a positive integer, x is prime, x < 20} and B = {x : x is a positive integer, x is odd, x < 20}. Find (i) A ∩ B (ii) n(A ∩ B).

Answer Key

Section A

1(i). A = {3,6,9,12,15,18,21,24}. 1(ii). B = {5,10,15,20,25}. 1(iii). A ∩ B = {15}. 1(iv). A ∪ B = {3,5,6,9,10,12,15,18,20,21,24,25}. 1(v). A′ = {1,2,4,5,7,8,10,11,13,14,16,17,19,20,22,23,25}

2(i). P ∩ Q = {3,5,7,11,13}. 2(ii). Q ∩ R = { } (disjoint). 2(iii). P ∪ R = {2,3,4,5,6,7,8,10,11,12,13,14}. 2(iv). (P ∩ Q)′ = {1,2,4,6,8,9,10,12,14,15}. 2(v). P ∪ Q ∪ R = {1,...,15}, n = 15

3(i). Mathematics only = 25 − 12 = 13. 3(ii). Physics only = 20 − 12 = 8. 3(iii). Total taking at least one = 13+8+12 = 33, so neither = 40 − 33 = 7

4(i). X ∩ Y = {x : 0 < x ≤ 10} = {1,...,10}. 4(ii). X ∪ Y = {x : −5 ≤ x ≤ 12}. 4(iii). X′ = {x : x < −5 or x ≥ 10}

5. A (factors of 24) = {1,2,3,4,6,8,12,24}, B (factors of 36) = {1,2,3,4,6,9,12,18,36}. 5(i). A ∩ B = {1,2,3,4,6,12}. 5(ii). A ∪ B = {1,2,3,4,6,8,9,12,18,24,36}. 5(iii). n(A ∩ B) = 6

6(i). n(A ∪ B) = 28 + 25 − 15 = 38. 6(ii). Neither = n(ε) − n(A ∪ B) = 50 − 38 = 12

7. $n(P \cap Q) = n(P) + n(Q) - n(P \cup Q) = 20 + 15 - 28 = 7$. Since $n(P \cap Q) \neq 0$, P and Q are not disjoint — they share 7 common elements

Section B

1(i). $X = \{1,\ldots,11\}$, $Y = \{2,4,\ldots,20\}$. $X \cap Y = \{2,4,6,8,10\}$.   1(ii). $X \cup Y = \{1,2,\ldots,11,12,14,16,18,20\}$.   1(iii). $X' = \{x : x \leq 0 \text{ or } x \geq 12\}$

2(i). $A = \{-3,-2,-1,0,1,2,3,4,5,6,7\}$, B = positive integers. $A \cap B = \{1,2,3,4,5,6,7\}$.   2(ii). $n(A \cap B) = 7$

3(i). $P = \{1,\ldots,9\}$, $Q = \{3,6,9,12,15\}$. $P \cap Q = \{3,6,9\}$.   3(ii). $P \cap Q' = \{1,2,4,5,7,8\}$.   3(iii). $n(P \cap Q') = 6$

4(i). $\mu = \{-10,\ldots,10\}$, $R = \{1,\ldots,10\}$. $R' = \{-10,-9,\ldots,0\}$.   4(ii). $n(R') = 11$

5(i). $A = \{4,8,12,16,20,24,28,32,36,40\}$, $B = \{6,12,18,24,30,36\}$. $A \cap B = \{12,24,36\}$.   5(ii). $n(A \cap B) = 3$

6. Positive integers with $x^2 \leq 50$: $7^2 = 49 \leq 50$, $8^2 = 64 > 50$. $P = \{1,2,3,4,5,6,7\}$, $n(P) = 7$

7(i). A (primes < 20) = $\{2,3,5,7,11,13,17,19\}$. B (odd positive < 20) = $\{1,3,5,7,9,11,13,15,17,19\}$. $A \cap B = \{3,5,7,11,13,17,19\}$ (all primes except 2).   7(ii). $n(A \cap B) = 7$