

Topic: Three-Set (3-Dimensional) Venn Diagrams — Word Problems

Section A: General Three-Set Word Problems

1. In a survey of 200 students, 90 offer Mathematics (M), 80 offer Physics (P) and 70 offer Chemistry (C). 35 offer Mathematics and Physics, 30 offer Physics and Chemistry, 25 offer Mathematics and Chemistry, and 15 offer all three subjects. (a) Draw a Venn diagram to illustrate this information. (b) Find the number of students who offer (i) Mathematics only (ii) exactly two of the three subjects (iii) at least one of the three subjects (iv) none of the three subjects.
2. Out of 250 students, 120 play Football (F), 100 play Basketball (B), and 90 play Volleyball (V). 50 play Football and Basketball, 40 play Basketball and Volleyball, 45 play Football and Volleyball, and 20 play all three sports. (a) Draw a Venn diagram to illustrate this information. (b) Find the number of students who play (i) Volleyball only (ii) exactly one sport (iii) none of the three sports.
3. In a class of 150 students, the number who speak only English is 30, only French is 20, only Yoruba is 15, English and French only is 10, French and Yoruba only is 8, English and Yoruba only is 12, and all three languages is 5. (a) Draw a Venn diagram to illustrate this information. (b) Find (i) the number who speak at least one language (ii) the number who speak none of the three languages (iii) the total number who speak French.

Section B: Questions Involving Only Two Sets (A and B) Out of Three (A, B and C)

In each question below, three sets are surveyed, but you are only asked to find information relating to two of the three sets.

- 1. In a class of 120 students, 65 offer Mathematics (M), 50 offer Physics (P), and 45 offer Chemistry (C). 20 offer Mathematics and Physics, 15 offer Physics and Chemistry, 18 offer Mathematics and Chemistry, and 8 offer all three subjects. (a) Draw a Venn diagram to illustrate this information. (b) Using your diagram, find (i) the number who offer Mathematics and Physics only (not Chemistry) (ii) $n(M \cup P)$ (iii) the number who offer Physics but not Mathematics.**
- 2. In a survey of 160 students on sport, 80 play Football (F), 70 play Basketball (B), and 40 play Hockey (H). $n(F \cap B) = 30$, $n(B \cap H) = 15$, $n(F \cap H) = 20$, and $n(F \cap B \cap H) = 10$. (a) Draw a Venn diagram to illustrate this information. (b) Find (i) the number who play both Football and Basketball only (excluding Hockey) (ii) $n(F \cup B)$ (iii) $n(F \cap B)$, regardless of Hockey.**
- 3. In a survey of 100 shoppers, 55 bought Rice (R), 40 bought Beans (Be), and 35 bought Garri (G). $n(R \cap Be) = 20$, $n(Be \cap G) = 15$, $n(R \cap G) = 18$, and $n(R \cap Be \cap G) = 8$. (a) Draw a Venn diagram to illustrate this information. (b) Find (i) the number who bought both Rice and Beans only (not Garri) (ii) $n(R \cup Be)$ (iii) the number who bought Rice but not Beans.**

Section C: Further WAEC-Style Questions (Algebra & Shading)

1. In a class of 50 students, the number who like only Mathematics is x , only English is $2x$, only Yoruba is $(x + 2)$, Mathematics and English only is 3, English and Yoruba only is 5, Mathematics and Yoruba only is 4, all three subjects is 2, and none of the subjects is 6. (a) Draw a Venn diagram to illustrate this information. (b) Find (i) the value of x (ii) the number of students who like only Mathematics.

2. In a class of 100 students, x offer Mathematics only, x offer Physics only, $2x$ offer Chemistry only, 10 offer Mathematics and Physics only, 8 offer Physics and Chemistry only, 6 offer Mathematics and Chemistry only, 4 offer all three subjects, and 12 offer none of the subjects. (a) Draw a Venn diagram to illustrate this information. (b) Find (i) the value of x (ii) the number who offer Mathematics only (iii) the number who offer Chemistry only.

3. Three sets A, B and C are represented by three overlapping circles inside a universal set ϵ . Draw separate diagrams and shade the region representing (i) $A \cap B \cap C$ (ii) $(A \cup B \cup C)'$ (iii) $A \cap (B \cup C)'$ (iv) $A' \cap B' \cap C$.

Answer Key

Section A

1(b)(i). M only = $90 - (35 + 25 - 15) = 90 - 45 = 45$. (ii) Exactly two = $(35 - 15) + (30 - 15) + (25 - 15) = 20 + 15 + 10 = 45$. (iii) At least one = $90 + 80 + 70 - (35 + 30 + 25) + 15 = 240 - 90 + 15 = 165$. (iv) None = $200 - 165 = 35$

2(b)(i). Volleyball only = $90 - (40 + 45 - 20) = 90 - 65 = 25$. (ii) Football only = $120 - (50 + 45 - 20) = 45$; Basketball only = $100 - (50 + 40 - 20) = 30$; exactly one = $45 + 30 + 25 = 100$. (iii) At least one = $120 + 100 + 90 - (50 + 40 + 45) + 20 = 310 - 135 + 20 = 195$; none = $250 - 195 = 55$

3(b)(i). At least one = $30 + 20 + 15 + 10 + 8 + 12 + 5 = 100$. (ii) None = $150 - 100 = 50$. (iii) French (only + all overlaps involving French) = $20 + 10 + 8 + 5 = 43$

Section B

1(b)(i). $M \cap P$ only = $20 - 8 = 12$. (ii) $n(M \cup P) = 65 + 50 - 20 = 95$. (iii) Physics but not Mathematics = $P - (M \cap P) = 50 - 20 = 30$

2(b)(i). $F \cap B$ only (excluding H) = $30 - 10 = 20$. (ii) $n(F \cup B) = 80 + 70 - 30 = 120$. (iii) $n(F \cap B)$ regardless of Hockey = 30 (this figure already includes the triple overlap)

3(b)(i). $R \cap Be$ only (excluding G) = $20 - 8 = 12$. (ii) $n(R \cup Be) = 55 + 40 - 20 = 75$. (iii) Rice but not Beans = $R - (R \cap Be) = 55 - 20 = 35$

Section C

1(b). Sum of all regions = 50: $x + 2x + (x + 2) + 3 + 5 + 4 + 2 + 6 = 50 \rightarrow 4x + 22 = 50 \rightarrow 4x = 28 \rightarrow x = 7$. (ii) Only Mathematics = $x = 7$

2(b). Sum of all regions = 100: $x + x + 2x + 10 + 8 + 6 + 4 + 12 = 100 \rightarrow 4x + 40 = 100 \rightarrow 4x = 60 \rightarrow x = 15$. (ii) Mathematics only = $x = 15$. (iii) Chemistry only = $2x = 30$

3. (i) $A \cap B \cap C'$ is the part where A and B overlap, but outside C. (ii) $(A \cup B \cup C)'$ is the region outside all three circles, within ε . (iii) $A \cap (B \cup C)'$ is the part of A that does not overlap with B or C at all — i.e. A only. (iv) $A' \cap B' \cap C$ is the part of C that does not overlap with A or B — i.e. C only.