

Topic: Venn Diagrams — Representing Sets and Solving Set Operations (Union, Intersection, Complement)

Part A: Two-Set Venn Diagrams

1. The sets $P = \{1,2,3,4,6,8,12,24\}$ and $Q = \{1,2,3,4,6,9,12,18,36\}$ are subsets of $\epsilon = \{1,2,3,\dots,40\}$. (a) Draw a Venn diagram to illustrate this information. (b) Use your diagram to find (i) $P \cap Q$ (ii) $P \cup Q$ (iii) $(P \cup Q)'$.

2. The universal set $\epsilon = \{x : x \text{ is an integer, } 1 \leq x \leq 20\}$. $X = \{x : x \text{ is a multiple of } 3\}$ and $Y = \{x : x \text{ is a multiple of } 4\}$. (a) List the elements of X and Y . (b) Draw a Venn diagram to represent ϵ , X and Y . (c) Find (i) $X \cap Y$ (ii) $X \cup Y$ (iii) $(X \cup Y)'$.

Part B: Three-Set Venn Diagrams

1. The sets $P = \{1,2,3,4,6,8,12,24\}$, $Q = \{1,2,3,4,6,9,12,18,36\}$ and $R = \{1,2,4,8,16,32\}$ are subsets of $\epsilon = \{1,2,3,\dots,40\}$. (a) Draw a Venn diagram to illustrate this information. (b) Use your diagram to find (i) $P \cap Q \cap R$ (ii) $P \cap (Q \cup R)$ (iii) $(P \cup Q)'$.

2. The universal set $\epsilon = \{x : x \text{ is an integer, } 1 \leq x \leq 20\}$. $X = \{x : x \text{ is a multiple of } 3\}$, $Y = \{x : x \text{ is a multiple of } 4\}$ and $Z = \{x : x \text{ is a factor of } 24\}$. (a) List the elements of X , Y and Z . (b) Draw a Venn diagram to represent ϵ , X , Y and Z . (c) Find (i) $X \cap Y$ (ii) $Y \cap Z$ (iii) $X \cap Z$ (iv) $X \cap Y \cap Z$.

Answer Key

Part A: Two-Set Venn Diagrams

1. P (factors of 24) = $\{1,2,3,4,6,8,12,24\}$, Q (factors of 36) = $\{1,2,3,4,6,9,12,18,36\}$. (i) $P \cap Q = \{1,2,3,4,6,12\}$. (ii) $P \cup Q = \{1,2,3,4,6,8,9,12,18,24,36\}$. (iii) $(P \cup Q)' = \epsilon - (P \cup Q) = \{5,7,10,11,13,14,15,16,17,19,20,21,22,23,25,26,27,28,29,30,31,32,33,34,35,37,38,39,40\}$

2(a). X (multiples of 3, 1-20) = $\{3,6,9,12,15,18\}$. Y (multiples of 4, 1-20) = $\{4,8,12,16,20\}$. (c)(i) $X \cap Y = \{12\}$. (ii) $X \cup Y = \{3,4,6,8,9,12,15,16,18,20\}$. (iii) $(X \cup Y)' = \{1,2,5,7,10,11,13,14,17,19\}$

Part B: Three-Set Venn Diagrams

1. P (factors of 24) = $\{1,2,3,4,6,8,12,24\}$, Q (factors of 36) = $\{1,2,3,4,6,9,12,18,36\}$, R (powers of 2 up to 32) = $\{1,2,4,8,16,32\}$. (i) $P \cap Q \cap R = \{1,2,4\}$. (ii) $Q \cup R = \{1,2,3,4,6,8,9,12,16,18,32,36\}$; $P \cap (Q \cup R) = \{1,2,3,4,6,8,12,24\} \cap (Q \cup R) = \{1,2,3,4,6,8,12\}$. (iii) $P \cup Q = \{1,2,3,4,6,8,9,12,18,24,36\}$; $(P \cup Q)' = \epsilon - (P \cup Q) = \{5,7,10,11,13,14,15,16,17,19,20,21,22,23,25,\dots,40\}$ (all elements of ϵ from 1-40 not listed in $P \cup Q$).

2(a). X (multiples of 3, 1-20) = $\{3, 6, 9, 12, 15, 18\}$. Y (multiples of 4, 1-20) = $\{4, 8, 12, 16, 20\}$. Z (factors of 24, 1-20) = $\{1, 2, 3, 4, 6, 8, 12\}$. (c)(i) $X \cap Y = \{12\}$. (ii) $Y \cap Z = \{4, 8, 12\}$. (iii) $X \cap Z = \{3, 6, 12\}$. (iv) $X \cap Y \cap Z = \{12\}$